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The Sensitivity Analysis of Service and Waiting Costs of A Multi Server Queuing Model

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ABSTRACT: Queuing theory deals with waiting lines and queues are usually appear at the service providers. Some results regarding the probable number of customers, the probable waiting time in the system and in the queue, service and waiting costs can be obtained by applying multi server queuing model. This research article explores the sensitivity analysis between expected waiting cost of consumer and expected service cost of server, and total expected cost of the multi-server queuing model.

Key Words: M/M/S Queuing model, waiting lines, Number of servers, expected waiting cost, expected service cost, Service provider, Consumer, Sensitivity analysis.

1. INTRODUCTION

If the waiting lines are lengthy than the fruitful environment among consumer along lines will not be generated; besides the lengthy queues lead to a loss to consumer as well as service provider. The service provider with low standards and least expenditure causes the danger of high dissatisfaction in the minds of consumers and there will be damage in growth of the future business. In contrast if the service provider offers redundant service to the customers than there will be a chance of getting dissatisfaction of service cost. The waiting cost and the service cost are the basic costs which play an important role in running a system without any disturbance.

In 2018, Vijay Prasad et. al., in their research article found the required number of servers and expected number of customers in the system of multi server queuing model by using LPP- graphical method. T Srinivasa Rao et.al., in 2018, in their research paper examined a two- stage queuing system where the arrivals are Poisson with rate depends on the condition of the server to specific; vacation, pre-service, operational or break down state. V Vasantha Kumar et.al., in 2016, in their paper studied a $M/E_k/1$ queuing model where the service is provided in two-phases one by one in succession. In 2016, S Hanumantha Rao et.al., in their research article analyzed an N-policy, two phase queuing system where the service station is subject to break down while in operation and repair may delay due to non-availability of the repair facility.This research article traces the minimum number of required servers by applying multi server queuing model in which waiting cost and service cost are computed and they are put under sensitivity analysis.

2. SYMBOLS AND NOTATIONS

The basic indexes of the queuing systems

γ= Average rate of arrival
κ= Average rate of service
N_q= Probable number of consumers in the waiting line
N_s = Probable number of consumers in the system
T_q= Probable waiting time of a consumer in the waiting line
T_s= Probable waiting time of a consumer in the system

The performance measures of $(M / M / S / \infty / FCFS)$ model

The probable number of consumers in waiting line

$$N_q = \left[\frac{1}{(s-1)!} \left(\gamma \big/ \kappa \right)^s \left(\frac{\gamma\kappa}{(s\kappa - \gamma)^2} \right) \right] P_0$$

Where $P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\gamma \big/ \kappa \right)^n + \frac{1}{s!} \left(\gamma \big/ \kappa \right)^s \left(\frac{s\kappa}{s\kappa - \lambda} \right) \right]^{-1}$

The probable number of consumers in the system

$$N_s = N_q + \frac{\gamma}{\kappa}$$

The probable waiting time of a consumer in the waiting line

$$T_q = \frac{N_q}{\gamma}$$

The probable waiting time of a customer in the system

$$T_s = T_q + \frac{1}{\kappa}$$

Mathematical expectation of service cost of multi-server model $E(S_c) = C_s * S$

Expected waiting cost in the system $E(W_c) = C_w * N_s$

The total expected cost $E(T_c) = C_w * N_s + C_s * S$

Description	Symbol
Number of server	S
Each server's service cost	C_s
Each consumer waiting cost	C_w

3. Results and Discussion

To make the sensitivity analysis between service and waiting costs in a multi server model one can consider the average rate of arrival ($\gamma = 10$), the average rate of service ($\kappa = 5$). The fundamental principle for the existence of system is

$$\frac{\gamma}{S\kappa} < 1 \quad \text{(or)} \quad S > \frac{\gamma}{\kappa} \; i.e. \; S>2$$

The performance measures of $(M / M / S / \infty / FCFS)$ model depicted below

Table1: Performance measure (N_s) of multi server queuing model

Number of servers (S)	N_s
3	2.89
4	2.17
5	2.04
6	2.01
7	2

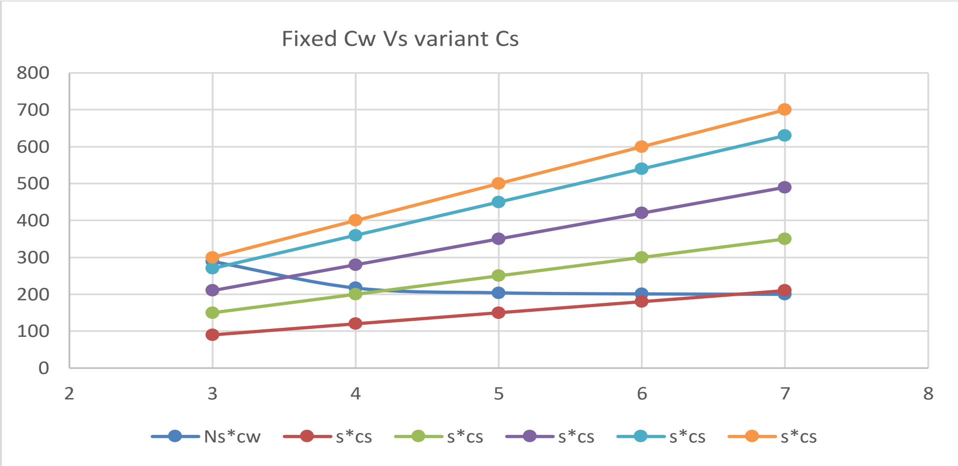
3.1 Sensitivity Analysis of service and waiting costs in multi server model

Case – I

In this case the waiting cost of each consumer has taken one fixed value and service cost of each server has taken different values in increasing order. The expected waiting cost of cosumer and the expected service cost of the system are computed in each case of multi server queuing model.

Table 2: Fixed waiting cost of each consumer and variant service cost of each server.

s	Ns	Ns*cw	s*cs	s*cs	s*cs	s*cs	s*cs
3	2.89	289	90	150	210	270	300
4	2.17	217	120	200	280	360	400
5	2.04	204	150	250	350	450	500
6	2.01	201	180	300	420	540	600
7	2	200	210	350	490	630	700



Graph 1: Fixed waiting cost of each consumer and variant service cost of each server.

The graph has been ploted with the number of servers verses expected cost and from the graph one clearly observe that when the service cost of each server is less then the expected service cost of the system is also less. As the service cost of each server increases gradually then the expected service cost alsoincreases gradually. When the expected service cost is less then the expected waiting cost of the consumer is also less but more number of servers are needed.Hence service provider has to provide more number of servers to optimize the system. As expected serivce cost of the system increases gradually then the expected waiting cost also increases gradually which indicates that if service provider provides quality service (expensive) then consumers willing to stay in the waiting line even though expected waiting cost is high. From this phenomenon one can conclude that the consumers have high interest to continue in the system as in following two cases:

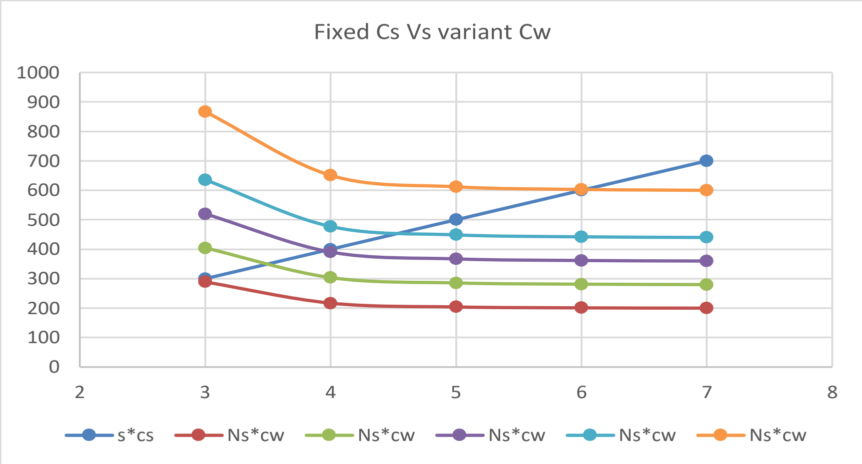
i. If the service provider provides good quality of service
ii. If the service provider provides more number of servers in the system.

Case-II

In this case the service cost of each server has taken one fixed value and waiting cost of each consumer has taken different values in increasing order. The expected waiting cost of cosumer and the expected service cost of the system are computed in each case of multi server queuing model.

Table 3: Fixed service cost of each server and variant waiting cost of each consumer.

s	Ns	s*cs	Ns*cw	Ns*cw	Ns*cw	Ns*cw	Ns*cw
3	2.89	300	289	404.6	520.2	635.8	867
4	2.17	400	217	303.8	390.6	477.4	651
5	2.04	500	204	285.6	367.2	448.8	612
6	2.01	600	201	281.4	361.8	442.2	603
7	2	700	200	280	360	440	600



Graph 2: Fixed service cost of each server and variant waiting cost of each consumer.

The graph has been ploted with the number of servers verses expected cost and from the graph one clearly observe that when the waiting cost of each consumer is small then the expected waiting cost of the system is also small. As the waiting cost of each consumerincreases gradually then the expected waiting cost alsoincreases gradually. When the expected waiting cost is low then the expected service cost of the system is also low but the number of servers are small. Hence service provider has to provide less number of servers to optimize the system. As the expected waiting cost of the system increases gradually then the service cost also increases gradually which indicates that as service provider provide quality service (expensive) then consumers are willing to stay in the waiting line even though expected waiting cost is high. From this phenomenon one can conclud that the service providerhave interest to provide the service and this can be seen in the following two cases:

i. The service provider will provide the good quality of service if the consumer offers high waiting cost to service provider.
ii. The service provider will provide more number of serves If the consumers are willing to continue in the system.

4. Conclusions

This research article traces the minimum number of required servers by applying multi server queuing model in which waiting cost and service cost are computed and they are put under sensitivity analysis. The consumers have high interest to continue in the system if the service provider provides good quality of service or the service provider provides more number of servers in the system. The service provider will provide the good quality of service if the consumer offers high waiting cost to service provider. Furthermore in the above discussions it has been established that the service provider will provide more number of serves if the consumers are willing to continue in the system.

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